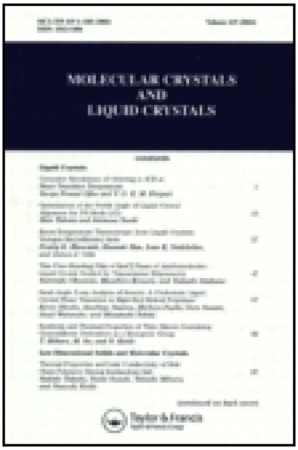

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# Analysis of Voltage-Controlled-Twist Effect using a Generalized Two-Constant Anchoring Energy

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# Analysis of Voltage-Controlled-Twist Effect using a Generalized Two-Constant Anchoring Energy

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*Abstract* In a previous work, via a second-order sphericalharmonic expansion, we built a generalized anchoring energy including two coupling constants. In the present paper, using this energy, we analyze the voltage-controlled-twist (VCT) effect [G.P. Bryan-Brown et al., *Nature (London)*, **392**, 365 (1998)]. Our analytical results indicate explicitly the presence of the twist threshold and saturation field, and demonstrate the condition of the occurrence of the VCT effect

*Keywords:* two-constant anchoring energy; voltage-controlled-twist effect; homeotropic substrate with in-plane anisotropy

## INTRODUCTION

Surface anchoring is a crucial point in the science and technology of liquid crystals. Over the past several decades, researchers have invented many kinds of methods of surface treatment [1,2], and made considerable efforts to build an appropriate formalism of the anisotropic surface energy. In the early stage, the concept of strong anchoring was employed which assumes that the substrate imposes a specific orientation (the easy axis) at the boundary of liquid crystals. As a suitable improvement, the concept of weak anchoring is suggested which permits the surface director to deviate from the easy axis, with the cost of finite surface energy depicted by a Taylor-type formula [3]. Rapini and Papoular (RP) [4] suggested a harmonic expression for the interfacial energy of homeotropic anchoring per unit area: $g_s = C\sin^2\Phi_0$, where $\Phi_0$ is the polar angle of the director at the surface, and the coefficient $C$ is termed anchoring strength or anchoring energy. The RP expression has attracted extensive attentions, and many followers have taken attempts to generalize it in order to describe the planar and tilt anchorage [5-8]. It is noteworthy that Beica et al. [8] constructed a formula including two coupling constants. However, their expression is still incomplete, partly due to its incapability of giving a suitable description on the homeotropic surface with in-plane anisotropy [9,10], and partly due to its difficulty of including the nontriviality in unidirectional planar anchoring case revealed by Brown et al. using a finite-element method [11]. Some researchers suggested that the anchoring energy might be separated into two parts, a polar moiety and an azimuthal moiety [12]. Its lowest-order approximation is sometimes used for the analysis of observation [10].

In a recent work [13], we built a two-constant formula through a second-order spherical-harmonic expansion of the surface energy. We found that the formulas used in Refs. [4-8] are all specific cases of this expression. However, those used in Refs. [10] and [12] are not in line with our result. It is instructive to demonstrate an example to which the present formula is applied whereas the others [4-8, 10] are ineffective. We find that the voltage-controlled-twist (VCT) effect [9] is a suitable topic. This effect is based on the frustrated alignment of a liquid crystal film with negative dielectric anisotropy, sandwiched between a homeotropic substrate with in-plane grooving and a unidirectional planar anchoring surface. The grooving homeotropic substrate, in coarse-grained viewpoint [14], offers a boundary condition that the anchoring energies in the surface normal, the grooving axis, and their orthogonal direction are distinct from each other. Hence, it imposes a twist torque on the surface director if it is in tilt or in-plane orientation. As applying to the grooving homeotropic substrate, our anchoring energy gives

$$g_s = W_1 \cos^2\theta^0 \cos^2\phi^0 + W_2 \cos^2\theta^0 \sin^2\phi^0, \tag{1}$$

where $\theta^0$ and $\phi^0$ are the tilt and azimuthal angles of the boundary director. $W_1$ and $W_2$ ($W_1 > W_2 > 0$) are the two anchoring constants corresponding to the deviation of the director from $z$-axis in $xz$ and $yz$ plane, respectively. It should be noted that in homeotropic case the anchoring energies used in Refs. [4-8] degenerate into single constant form and hence are difficult to be applied to the investigation of the VCT effect. The expression used in Ref. [10] has an unphysical azimuthal dependence in vertical alignment. As a result, the orientation with $\phi^0=\pi/2$ is strong preferred in vertical alignment ($\theta^0=\pi/2$). This yields at once the decision that the VCT threshold field is exactly zero, which is in complete contradiction with the experimental results. In this paper, we devote ourselves to analyze the VCT effect using Eq. (1), and build the analytical expression of the threshold and saturation field.

## ANALYSIS

The total free energy per unit area is written as

$$F=\int_0^l dz\frac{1}{2}[k_{11}(\nabla\cdot\vec{n})^2+k_{22}(\vec{n}\cdot\nabla\times\vec{n})^2+k_{33}(\vec{n}\times\nabla\times\vec{n})^2-\vec{D}\cdot\vec{E}]+g_s^- \qquad (2)$$

where $\vec{n}=(\cos\theta\cos\phi,\cos\theta\sin\phi,\sin\theta)$ is the director. $g_s^-$ is shown in Eq. (1). A rigid unidirectional planar boundary condition is assumed for the opposite substrate.

This free energy can be treated with normal steps. For reasons of simplicity, a single-elastic-constant approximation $k_{11}=k_{22}=k_{33}=k$ is employed. We also use a simplified electric energy $-\vec{D}\cdot\vec{E}\approx-\varepsilon_a E^2=\varepsilon_a^+ E^2$, where $\varepsilon_a$ is the negative dielectric anisotropy. The equilibrium equations are attained through variational calculation of Eq. (2) in terms of $\theta$ and $\phi$, and finally the analytical solution is obtained. To save space, we show only the results here [14]. We define $\lambda=\pi k/(2lW_1)$, $\gamma=(W_1-W_2)/W_1$, $u'=E_{th}/E_c$, and $u''=E_s/E_c$, where

$E_c = \pi(k/\varepsilon_a^+)^{1/2}/l$. The threshold field $u'$ and the threshold tilt $\theta_{th}^0$ are given by Eqs. (3)-(5),

$$\Lambda = 2\lambda u' / (\sin^2 2\theta_{th}^0 + 4\lambda^2 u'^2 \cos^2 \theta_{th}^0)^{1/2}, \tag{3}$$

$$\pi u'/\Lambda = K(\Lambda) - F(\Lambda, \pi/2 - \theta_{th}^0), \tag{4}$$

$$\frac{\pi u'}{\Lambda} - \frac{\lambda u'}{\gamma\Lambda\cos^2\theta_{th}^0} + \frac{\Lambda\sin^2\theta_{th}^0}{\lambda u'} = E(\Lambda) - E(\Lambda, \pi/2 - \theta_{th}^0), \tag{5}$$

in which $F$, $E$ and $K$ denote the elliptical integrals of the first and second kinds, and the corresponding complete forms [15]. $\Lambda$ is a subsidiary variable. The saturation field $u''$ and the maximal twist angle $\phi_s^0$ are defined by Eqs. (6)-(8),

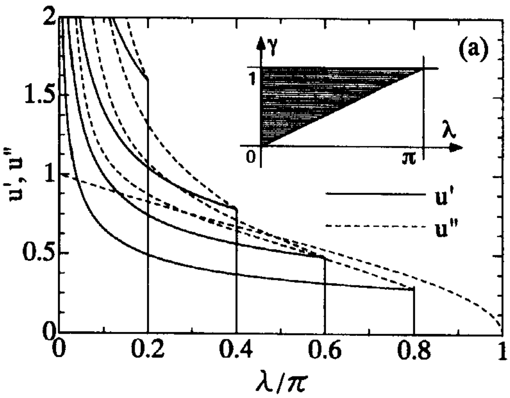
$$2\lambda\phi_s^0 = \pi\gamma \sin 2\phi_s^0, \tag{6}$$

$$\lambda Z = (1 - \gamma \sin^2 \phi_s^0) \tanh(\pi Z), \tag{7}$$

$$u''^2 = Z^2 + \gamma^2 \sin^2 2\phi_s^0 / \lambda^2. \tag{8}$$

Here $Z$ is a subsidiary variable.

We find that if the VCT effect exists, the parameters $(\lambda, \gamma)$ must locate in the effective triangular area (ETA) shown in the inset of Fig. 1(a). The results about the threshold and saturation properties are demonstrated in Fig. 1, in which $u'$, $u''$, $\theta_{th}^0$, and $\phi_s^0$ are plotted as functions of $\lambda$, with constant parameter $\gamma$=0.2, 0.4, 0.6, 0.8, and 1.0, respectively. The lower right extremity of each curve corresponds to a point on the $\lambda = \pi\gamma$ boundary line of the ETA. Along this boundary line, $u' = u''$, and $\theta_{th}^0 = \phi_s^0 = 0$, i.e., a virtual twist occurs just as saturation is reached.



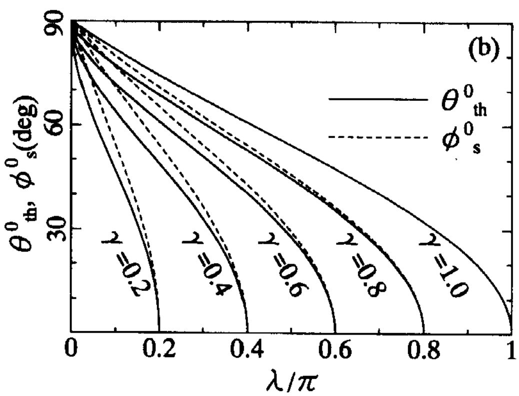




**Fig. 1** $\lambda$ and $\gamma$ dependence of the (a) threshold ($u'$) and saturation ($u''$) fields and (b) threshold tilt ($\theta_{th}^0$) and saturation twist ($\phi_s^0$) angles. For each curve, its constant parameter $\gamma$ is the same as the abscissa of its lower right extremity. The inset in (a) shows the effective triangular area (dashed) in which the VCT effect exists.

In Fig. 1(a), from each of the extremities, two curves

**originate, referring to the threshold and saturation fields, respectively. In Fig. 1(b), the two curves originating from each extremity demonstrate the threshold tilt and saturation twist, respectively. It is interesting that the former is always approximate to, although a little less than, the latter. It should be pointed out that for $\gamma$=1.0, corresponding to the upper boundary of the ETA, it holds that $u' = 0$, and $\theta_{th}^0=\phi_s^0$.**

**Summarily, we have analyzed the VCT effect using the two-constant anchoring energy. It is found that for the existence of this effect, the parameters ($\lambda$,$\gamma$) must be located in a triangular area. The threshold and saturation properties have been demonstrated. This may be beneficial for the understanding and further exploration in relevant fields.**

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